Section 1.4 - Euler's Method
First, realize the following:

EULER'S METHOD IS STRAIGHTFORWARD!

For example, if our differential equation is

$$\frac{dy}{dt} = y^2+t$$

and our initial point is t=0, y=1, then dy/dt at this point is

$$\frac{dy}{dt} = 1^2+0=1$$

The equation of the line through this point is 1*(t-t0) = y-y0 with t0 = 0 and y0 = 1. So this equation is t = y - 1 or y = t + 1.

If t is increased by an increment of .1, the new value of t is 0.1 and we can calculate the new value of y as y = .1+1 = 1.1

The slope at the point t=.1, y=1.1 is dy/dt = 1.1*1.1 + .1 = 1.31

If we increase t by 0.1 again, the new value of t is 0.2 and
y - y1 = 1.31(.2 - .1) or
y - 1.1 = 1.31(.1) where y1 = 1.1

The new value of y is 1.31(.1) + 1.1 = 1.231
A pattern emerges.

(new y) = (old y) + (dy/dt at old point)*(change in t)

(new t) = (old t) + (change in t)

Note that the (change in t) quantity above is 0.1 in our example. This quantity (change in t) is called the "step size". The smaller the step size, the better the approximation.

THE FORMULAS FROM THE PREVIOUS PAGE MAY BE GIVEN AS

$$\boxed{y_{k+1} = y_k + f(t_k, y_k)\,\Delta t}$$

WHERE y_{k+1} = NEW y , y_k = OLD y

$f(t_k, y_k) = \frac{dy}{dt}$ AT OLD t & y VALUES

Δt = CHANGE IN t = STEP SIZE

ALSO,

$$\boxed{t_{k+1} = t_k + \Delta t}$$

WHERE t_{k+1} = NEW t

t_k = OLD t

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USING THESE FORMULAS, WE MAY GENERATE VALUES OF t & y.

k	t_k	y_k	$m_k = \frac{dy}{dt}$
0	0	1	$1^2 + 0 = 1$
1	0.1	$1 + (1)(.1) = 1.1$	$(1.1)^2 + .1 = 1.31$
2	0.2	$1.1 + 1.31(.1) = 1.231$	$(1.231)^2 + .2 = 1.715361$
3	0.3	$1.231 + (1.715361)(.1) = 1.4025361$	

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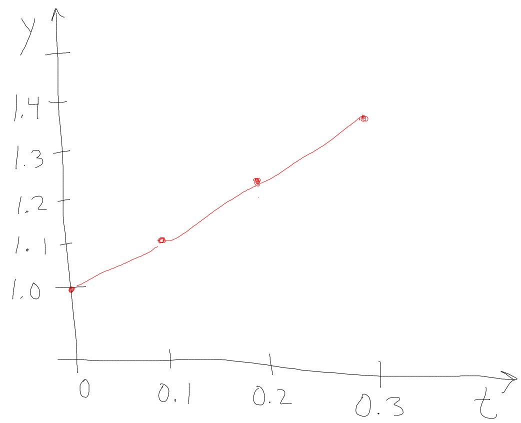
THE PLOT OF THIS DATA IS



As you compute successive values of y and t, you are generating the plot of the solution of the differential equation. The smaller the step size, the closer this generated approximation will be to the graph of the exact solution. Euler's method is one of several highly used numerical techniques. Graphmatica uses another numerical technique called "Runge-Kutta", which is an improved version on Euler's method.

All numerical techniques introduce error to some extent and sometimes the error is enough to greatly affect the outcome.

Do the exercises for Section 1.4.
Read the book!